PROBABILITY

MAT 4287
FALL 2020

Instructor. Andrew McIntyre. `amcintyre@bennington.edu`. I am not physically in my office this term. The preferred way to contact me is through the Bennington Math Slack (I will send you a link to sign up); you can also contact me by email.

Credits. 4 credits.

Class times and location. Tuesdays and Fridays, 8:30am–12:10pm. The course meets in the first 7 week block of the term. The first class is Friday, September 4, 2020, and the last class is Tuesday, October 15, 2020.

The class is conducted remotely and mostly synchronously (see below for details). There is an element of group work in the class; there will be some classroom space available during class hours for student groups who want to meet in person (masked and socially distanced) and who are capable of and feel safe doing so. The classroom space is in Dickinson 117, 209, 212, and 148. I will explain how to reserve these spaces for your groups after the first class.

Office hours. I will not set particular office hours this term. You can contact me any time, through Slack or email. During the term, I'll aim to always reply within 24 hours. I can either answer questions over Slack/email, or we can set up a Zoom meeting if you prefer.

Texts. The required text is

- William Feller, *An Introduction to Probability Theory and Its Applications*, Vol. 1, 3rd Edition, corrected printing, Wiley, 1970, ISBN 978-0471257080

The book is *very* expensive new, for which I apologize. It is a classic of the subject, and I did not find any good substitute. We will be working from the book quite intensively. I will be making the chapters we are working on available as pdfs, so you do not need to purchase the book. I'm sorry for that, it's nicer to have a physical book. (In particular, the book is a good reference for many topics we won't have time to get to.) You can sometimes find used copies of this book at more reasonable prices (try abebooks or alibris). However, be sure to get the 3d corrected edition; the other editions are quite different.

An alternate reference text for this class is

- Charles M. Grinstead and J. Laurie Snell, *Introduction to Probability*, 2nd Revised Edition, American Mathematical Society 2012, ISBN 978-0821894149

This book is open-source; you can find a free pdf at
`https://math.dartmouth.edu/~prob/prob/prob.pdf`.
This text is not required. I am including this book because it can sometimes be helpful to have an alternate explanation for topics.

Calculators and computers. A simple scientific calculator or calculator app will be sufficient for almost all the work in this class. You will not need a graphing calculator, though you are welcome to use one if that is what you already have and you are comfortable with it. I will occasionally ask you to graph something on a computer; I recommend the free web app `https://www.desmos.com/calculator`.

What this class covers. This first course in probability will take a classical approach, following the classic text by Will Feller, *An Introduction to Probability Theory and its Applications*. In particular, the topics will include: sample spaces, conditional probabilities, independence, random variables, expectation and variance; the binomial, poisson, and normal distributions; the law of large numbers and the central limit theorem; random walks; and Markov chains. The course will not cover measure theory or formal proofs, but there will be proofs at an appropriate level of rigor. The aim will be to get a deep understanding of the classical concepts, mostly in the discrete case. Students should be well set up to learn more on continuous distributions and Bayesian approaches in future courses if needed. The class should be of interest for both theoretical and applied purposes. The class will be a prerequisite for Machine Learning in Spring 2021.

Prerequisites. Some familiarity with the language of set theory, the binomial theorem, exponential and log functions, and infinite series will be helpful. If students have taken MAT 2410 *Logic, Proofs, Algebra, and Set Theory* and MAT 4133 *Calculus A*, they will be well-prepared, but students need not have taken those particular courses. I will make available "preamble assignments", which students can work on before the class begins to develop the necessary skills.

Work expected. In this class, I am putting an emphasis on reading and understanding the text. I will ask you to read the text, make notes, comments, and your own examples, and contribute to class discussion boards on the text and problems. You will do collaborative work on problems, and assignments that you will hand in. I also encourage you to spend time helping your classmates understand the material, and will explicitly credit you for that work as well.

The usual expected workload for a 4 credit class is 10 hours per week, meaning 4 hours per week in class and 6 hours outside of class. (The logic of this is that a full 16 credit schedule would then be 40 hours per week of work.) Because this class is 4 credits in 7 weeks, the expected workload is 20 hours per week, meaning 8 hours of designated class time, and 12 hours spent on work outside of class time.

You may not be spending fully 20 hours every week, but if you spend much less than that, it will be difficult to get full value from the class and to do well. On the other hand, I am not expecting more than 20 hours per week. If the class finds that the average student time is getting close to or above that number, then I will adjust the expectations to keep it under 20 hours for most people.

I recommend scheduling 20 hours per week, (including class time), and doing the best you can with the work in those 20 hours. If you do this, you will be successful in the class, even if you don't complete all problems.

I understand that life may interfere with regular commitment to the work, particularly this term. If you are having difficulty putting in the time, please let me know as soon as possible. I will do my best to be flexible and help you stay on track. I am committed to making success in this class possible for any student who can put in the time.

How to submit work. I will make a Google Drive folder for you to upload your work. I will expect you to upload your work on assignments. If you worked on problems but can't solve them, submit what you do have: partial solutions, strategies, ideas, or just a sentence trying to explain why you are stuck.

I encourage you to also upload notes and other evidence of your work. In addition, each week I would like you to write a short list of what you have been spending time on that week: reading the book, working with others, explaining things to others, and so on.

One way of doing this is I would recommend is to keep one bound notebook, in chronological "journal" format. Label each item ("assignment", "lecture notes", and so on), but just write in the order you are doing the work. Talk to yourself in this book—that helps a lot with your thinking, and it helps me to see what you are thinking. (e. g. "I spent two hours reading Chapter I, and made the notes above. The main thing I don't understand is…"). Then, you can just upload new pages of your journal/notebook as you complete them.

However, you don't have to do it that way; any method that allows you to stay organized, and allows me to see what work you are doing, is fine.

I am not focused on specific due dates. I will make recommended due dates for assignments, but each week what I want to see is the work that you have completed that week. If you don't get an assignment completed one week, you can continue submitting that work in following weeks until you have completed it. (Though it may sometimes be better to leave an assignment aside and concentrate on newer work; I'll make suggestions to you about that, see the following.)

A technical note: please get a scanning app for your phone that will allow you submit your work as pdfs. There are a number of free apps available. These apps have a number of algorithms that make the scanned material much more legible than simply taking a photo. Also, they allow you to scan work as multiple-page pdfs, which is much easier for me to read than having a list of single pages in the folder.

Please put the date of your submission in the file name; in fact, just using the date as the file name is often the easiest way for me to navigate your work. For example, a filename like "2020-09-07-2" for the second thing you uploaded on September 7 keeps things in clear order. However, you are welcome to organize your folder however you like, as long as it is easy for me to navigate and find your work.

Feedback and assessment. Each week, I will look over the work you have uploaded to the Google Drive folder. I will let you know the day I am doing this once we get the term started. After looking over your work, I will write a narrative comment, and send it to you by Slack.

Many of the problems have final answers given, or can be checked by consistency or other means. So I will be focusing less on checking answers, and more on more global issues. For example, I might make comments like "you need to show more of your reasoning on more problems", or "there are some numerical errors in the early questions, please check those again".

I am also asking you to write something at least once a week about how you are spending your time; for example, you might be spending a lot of time on reading, or working with others, that isn't evident in your written work.

I will also try to make suggestions about how you are spending your time, and about what will help you as you proceed. For example, I might make comments like "you don't need to write quite so much", or "you can skip some questions on this last assignment and concentrate on the current assignment", or "try to go back and clarify these particular concepts in the previous chapter, they are holding you back now".

You are welcome and encouraged to write back and make this a dialogue, about how you are doing, what is going well, and what you need to work on.

Evaluation. There will be a "midterm" evaluation (in roughly the fourth week), and a final evaluation. For both evaluations, I will write a narrative paragraph, and I will ask you to write a self-evaluation narrative paragraph. If you are requesting a grade, I will ask you to propose a grade and I will assign one, for both midterm and final.

The criteria I use for your evaluation will be the same that I ask you to use for your self-evaluation. It will include:

- whether you are spending sufficient time and attention on this class
- how thoroughly you are reading and understanding the text
- how thoroughly and correctly you are solving the problems
- how persistent you are with difficult problems
- how creative you are with challenging problems
- how clearly and carefully you are writing your reasoning
- how much of the work you are completing
- how well you are contributing to collaborative work
- how well you are contributing to class discussions (in whatever form)
- how well you are helping other students in the class, if applicable
- how consistent your presence in the class is (see below)

The most important criterion is the time and effort that you are putting into the class.

I expect that, in most cases, your self-evaluation and my evaluation will be in general agreement. If we disagree, we will compare records; I may ask you to show me work on problems, or answer questions about concepts, if we disagree on how completely you have solved or understood something.

I will be responsible for the final narrative evaluation and grade that goes on your transcript, but in all but exceptional cases we should have reached consensus by then.

Note that it is never too late to "make things up"; if you figure out problems or concepts several weeks later than we covered them, you can always submit work to the folder to show me your progress. I will only "count" how solid your understanding and work are in the end, not when you got there or how long it took. (That said, since this is a seven week class, be aware that you won't have much time to catch up if you get too far behind!)

Notebooks. It is very important to keep a clear and organized notebook (or equivalent system) for this class. You will need records of your work to be able to show me, and you will also need a clear way to refer to and build upon your own work.

I recommend keeping a bound notebook (like a composition notebook), for this class only. Write your thinking about the problems in a journal format. Also write

class notes, solutions to problems, and summaries. This will help keep your thinking in one place. Number the pages, and put a table of contents at the end.

You may prefer to have a different system: perhaps you could have a bound notebook for notes and journal work, and write problem sets up separately. Or you could use a looseleaf binder, or a system of folders. Whatever you choose, your work should be accessible, clearly written, and understandable to both me and you. Don't assume that you will remember what you were doing later—write your explanations down in full as well as your computations.

Note that, since the work will be submitted by scans, if you like, you can have your problem set solutions in the same bound notebook that you use for notes and thinking.

A note on collaboration. For much of this class, I will be expecting you to work with other students. This might seem like it would be unproductive, if neither of you know what is going on—"the blind leading the blind". But in fact it can be very useful to talk through your thinking with another person. You say what you are thinking, explain where you are stuck, the other person does the same; and in articulating your thinking and difficulties aloud, it becomes clearer where to go next to resolve the problems.

This is not unlike how research mathematicians work together. We don't have a teacher who knows the answer; perhaps no-one does. We throw out ideas, and then try to test them: e. g. "if that was true, then how would it work in this simple example?". We try to generate more examples to test ideas, and if we think we have a solution, we try to verify it from different directions.

A large amount of class time will be devoted to collaborative work. I would also encourage you to work with other students outside of class. This is particularly helpful when you are having trouble persisting with problems when you are stuck—having someone else there can really help you push through.

For this class, I want to go one step further: I want everyone to succeed together. I want to explicitly encourage you to help each other. It's not a good idea just to *tell* someone how to do a problem, because then they don't really learn much. But you can give hints, make suggestions, talk things through together, and try to explain concepts in different ways. If you are spending time helping other students, please let me know; I will explicitly include this in your evaluations.

All that said, it is also important to be able to do work on your own, and to be sure that you truly understand things yourself and are not merely following along. Here is a good rule of thumb: collaborate with other students verbally, and on whiteboards. But when it comes to write down the solution, do this on your own, without referring to what you wrote collectively. Write the solution from your own thinking, do not just copy.

A note on lecturing. A major emphasis in this class is to help you learn to read a mathematics text yourself, as much as possible. This will make you more able to learn more probability (and more math) independently in future, beyond the confines of this class.

For this reason, I will be avoiding lecturing on things which are contained in the text.

I will make written "lectures", which will often be largely reading guides for the text. I may give you examples or a short overview to start with. Then I will give

section-by-section suggestions (particularly for the early chapters) about how to read the text, and how to figure out the difficult points.

We will not spend the entire 4 hour scheduled meeting time in class together. I will be available on Zoom for the full 4 hour meeting time, but not in formal lecture. Typically I will expect something like this: we devote an hour or so to collaborative work and reading (with me available for questions), then an hour formal "lecture" (which will be mostly me addressing problems or tricky points which many people encountered, plus class discussion), then an hour of you working on problems in groups and me answering questions, with some breaks in between.

My view is that the class should primarily offer things that you could not get by simply working on your own. There are two elements to this: one is collaboration with others (see the note above). The other is feedback as you are working. I will get you to start on problems on your own. If you reach a point where you are stuck, I will use my experience to try to give the best possible hint or suggestion appropriate to the point you are stuck at. As much as possible, I will try to only make suggestions that *you could have thought of yourself.* In this way I hope to help you improve your ability to figure out this sort of material on your own.

Presence policy (including attendance). It will be important to have a consistent presence in this class. Particularly on the shortened schedule, if you are not present for even a short while, it will be difficult to catch up.

I understand, however, that there could be many issues which might sometimes make it difficult or impossible to attend our scheduled class sessions. Therefore, instead of an "attendance" policy, I am making a more broad "presence" policy. It is required that you maintain a consistent presence in this class through all 7 weeks. Not having a presence for one week or more (total) is grounds for concern, and could possibly affect your evaluation.

Presence includes

- attendance at scheduled classes
- contributing to class discussions, either during scheduled class time, or on Perusall or Slack
- participation in collaborative group work, either during scheduled class time or outside of it
- helping other students, either during scheduled class time, or through Slack
- contacting me with questions, or discussing problems and ideas with me

I do *not* expect that everyone is doing *all* of these things all the time. But you can make up for one with the other; for example, if you cannot make it to a scheduled class, be sure to find time to work with students in your group, to talk to me, and/or to participate in online discussions. I should be able to see evidence of your presence consistently every week. If I don't, I will bring that up in the weekly feedback.

Weekly schedule.

Class	Dates	Chapter and topic
1	Sep 4	I Sample Spaces
2	Sep 8	
3	Sep 11	V Conditional Probability and Independence
4	Sep 15	
5	Sep 18	VI Binomial and Poisson Distributions
6	Sep 22	
7	Sep 25	VII and X Normal Distribution and Central Limit Theorem
8	Sep 29	IX Random Variables, Expectation, Variance
9	Oct 2	
10	Oct 6	III and XIV Fluctuations in Coin Tossing and Random Walks
11	Oct 9	
12	Oct 13	XV Markov Chains

Classroom Inclusivity. Everyone is welcome in this classroom, and in mathematics. Please be kind and respectful to one another.

IF YOU ARE NERVOUS ABOUT BEING IN THIS CLASS, please know that, in my experience, everyone can do mathematics, and do it well. You may have had negative experiences with math, and you may have had assumptions made on your ability for invalid reasons. It's normal to be nervous. You can do this, and you will be supported here.

IF YOU FEEL CONFIDENT ABOUT BEING IN THIS CLASS, (you might belong to both categories!), please be kind and respectful to your classmates. Do not talk over them, or make it seem like they should know things that they don't. You will find, as the class goes on, that we are all struggling with math problems in our own way. We are all in this together.

IF YOU ARE MADE UNCOMFORTABLE IN THIS CLASS for any reason, please talk to me about it. (Aside from being uncomfortable about hard problems, but you can talk to me about that too!) I will do what I can to solve the issue. If I do not resolve the issue, or if I am the one making you uncomfortable, please talk to a college administrator (e. g., from The Office of Diversity & Inclusion, Student Life, or Academic Services) or other faculty member.

Here's the official statement: Bennington College is committed to fostering the intellectual growth of all students, and to creating a learning environment where human cultural diversity is valued and respected. To that end, in this course all students can expect a respectful, welcoming and inclusive environment. I hope that all students in this course will openly share their unique perspectives and, just as importantly, respect the perspectives, comments, and contributions made by every other student and guest that participates in this course during the term. If you feel that at any time that this goal is not being met, please don't hesitate to see me, or speak with a college administrator (e. g., from The Office of Diversity & Inclusion, Student Life, or Academic Services) to share your concern.

Academic Accommodations and Health Resources. I am happy to work with you on any needed accommodations. Please let me know the situation.

Here is the official statement: Bennington College provides reasonable accommodations to students with documented disabilities when such accommodations are requested and necessary to ensure equal access to College programs and facilities. If

you believe you are entitled to an accommodation speak with Katy Evans, the Academic Services and Accommodations Advisor, about any disability-related needs. If approved, you will receive a memo detailing your specific accommodations; it is your responsibility to provide me with the memo and discuss the implementation of accommodations. Note that I will not be aware of your needs if you do not share this memo with me. Accommodations are not retroactive, so the sooner we meet to discuss your needs, the better. Also, students experiencing mental and/or physical health challenges that are significantly impacting their academic work are encouraged to speak with their faculty advisor and member of Academic Services (`academicservices@bennington.edu` or 440-4400) about the impact and to connect with resources through health and psychological services (440-4426 or 440-4451).

Statement on Basic Needs. At Bennington College, we understand that basic needs (food, housing, and wellness) have a direct impact on academic performance, mental-emotional-physical health, professional development, and holistic success of our students. If you have a personal circumstance or need that will affect your learning or performance in this course, please let me or your faculty advisor know so that we can direct you to available resources to help support you during the term.