Explanation of this Problem Set

I promised you one more problem set, PS 6, which would focus on theory. I have been short on time to compose a good enough problem set, so I was going to give up and assign some problems from one of the recommended readings.

However, as I look more carefully at these texts, I don't much like any of the exercises. So I'm going to try a different suggested assignment.

The texts which I most like for theory actually do not have exercises! So, Problem Set 6 consists of the following: pick one (or more) of the topics that I list below, from the texts that I have attached. Read through that section; try to follow the proofs, and supply the missing details (both texts are pretty terse!); and provide a couple of concrete examples.

Note that, if you are seriously doing a problem from PS 5, you don't necessarily need to do one from PS 6. In summary, from PS 5 and PS 6 together, I would like you to aim to do ONE problem (from EITHER PS5 or PS6) in some depth, or perhaps TWO or THREE problems total (from EITHER PS5 or PS6) if you just do the most basic part of the problem.

I hope that's not too confusing!

Note that the three sections below are not in any order; pick whichever seems most appealing!

1. Topics from Sharipov, A Course of Linear Algebra and Multidimensional Geometry

The first text is by R. A. Sharipov, *A Course of Linear Algebra and Multidimensional Geometry*. It covers the topics of this class at a similar depth that I was aiming for, but more coherently and with more details! You could pick any place in this book to start, and refer back to previous sections as needed for definitions; if you take it slowly, it should be possible to jump in any place. (And feel free to use supplementary references as well!)

Possible topics could be:

- Develop things carefully from the beginning: read and work through Chapter I, Sections 1–4, providing extra details for proofs and examples.
- Quotient spaces: Chapter I, Section 7.
- The algebra of projection operators: Chapter II, Section 2.
- Eigenvectors more carefully than I did them: Chapter II, Section 3 and 4.
- The Jordan Canonical form: Chapter II, Section 7.
- Dual Spaces: Chapter III, Sections 1–4.
- Inner Products: Chapter IV, Sections 1–3.
- Self-Adjoint Operators: Chapter V, Sections 3 and 4.

2. Topics from Emil Artin, Galois Theory

The second text is more related to abstract algebra, to the theory of fields. But the first chapter is a very precise and short introduction to linear algebra. Possible topics could be:

- Most of linear algebra, in a condensed form: Chapter I, Sections A–E (ten pages!).
- The theory of determinants, done very clearly (in most books it is kind of confusing): Chapter I, Section F.

- The fundamental theory of fields and field extensions: Chapter II, Sections A–as far as you can manage (getting through A–C would be already very interesting!). Note that you do not need to know any theory of determinants in order to follow this section.
- The impossibility of having an algebraic formula for the roots of a general fifth degree polynomial: Chapter III, Section C. You could look there for the statement, and then work backwards to see how much of the proof you could figure out!

3. Topics from A. J. McConnell, Applications of Tensor Analysis

If you're interested to learn more about tensors, it will be fun to start reading this classic book from the beginning, and see how much makes sense to you! If you follow through slowly, you will find that you know much of the early material already, though it may be in a somewhat unfamiliar form. If you choose this topic, just start at the beginning and see how far you can get!