APPLICATIONS OF TENSOR ANALYSIS

(formerly titled: Applications of the Absolute Differential Calculus)

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